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Disturbance Observer Based Control for Nonlinear MAGLEV Suspension System

Jun Yang, Argyrios Zolotas, Wen-Hua Chen, Konstantinos Michail, and Shihua Li

Abstract—This paper investigates the disturbance rejection problem of nonlinear MAGnetic LEViation (MAGLEV) suspension system with “mismatching” disturbances. Here “mismatching” refers to the disturbances that enter the system via different channel to the control input. The disturbance referring in this paper is mainly on load variation and unmodeled nonlinear dynamics. By linearizing the nonlinear MAGLEV suspension model, a linear state-space disturbance observer (DOB) is designed to estimate the lumped “mismatching” disturbances. A new disturbance compensation control method based on the estimate of DOB is proposed to solve the disturbance attenuation problem. The efficacy of the proposed approach for rejecting given disturbance is illustrated via simulations on realistic track input.

I. INTRODUCTION

During the past few years, MAGnetic LEViation (MAGLEV) suspension system has become one of the most promising transportation systems [1]. Compared with conventional trains, the superiority of MAGLEV train lies in that the friction, mechanical losses, vibration and noise are reduced substantially since it replaces the wheels by electromagnets and levitates on the guideway and avoids mechanical contact with the rail [2].

However, MAGLEV suspensions are essentially nonlinear systems with lumped disturbances consisting of external disturbance and model uncertainties [3], [4]. The air gap between the rail and the electromagnet is the variable to be controlled. In addition, the air gap is highly affected by the lumped disturbances. For stability and performance, both control input and quantities such as deflection to deterministic track inputs and RMS values of acceleration etc. should be constrained to appropriate limits [5].

A number of control approaches for MAGLEV systems have been researched throughout the last two decades, including PI/PA(Phase Advance) control [5], sliding mode control [6], adaptive control [7], robust control [8], $H_{\infty}$ control [9], [10] and some other traditional methods [11], [12]. Note that most of the methods can not achieve the desired control performance in the presence of unknown external disturbance or model uncertainties. The reason is that they do not deal with disturbances or uncertainties directly [13].

Disturbance observer based control (DOBC) provides an alternative approach to handle disturbances. Disturbance observer (DOB) technique was originally proposed by Ohishi et al. [14] for a position servo system in the late of 1980s. During the last two decades, DOBC schemes for linear and nonlinear systems have been put forward and applied successfully in many practical areas, e.g., servo control system [15], [16], robotic system [17], [18], [19], hard disk drive system [20], missile system [21], grinding system [13], [22]. The superiority of DOBC lies in that it provides a “patch” to disturbances for the existing control design without significantly changing the nominal performance. Simply, the DOB is designed in such a way that operates only when the disturbance appears.

It is noted that, in previous work in the literature, the DOBC methods were only applicable to “matching” disturbances. Here “matching” means the disturbances act via the same channels as the control inputs. For “mismatching” case of disturbances, i.e., the disturbances act via different channels to the control inputs, the existing DOBC methods are not applicable. It should be pointed out that “mismatching” disturbances are usually met in practical applications. For example, in low altitude aircraft systems, the lumped disturbance torques caused by unmodeled dynamics, external winds, and parameter perturbations, etc., always affect the states directly rather than through the input channels. It should be pointed out that the disturbances and uncertainties in MAGLEV suspension system belong to the “mismatching” case.

In this work, to enhance the degree-of-accuracy from the point view of control, we propose a new disturbance observer based control scheme to solve the “mismatching” disturbance rejection problem in MAGLEV suspension system mainly for deterministic performance. As for our control design, the model uncertainties caused by parameter perturbation and unmodeled nonlinear dynamics are merged into disturbances. Thus the external disturbances together with the model uncertainties are regarded as a kind of lumped disturbance. A state-space disturbance observer is designed to estimate such lumped disturbance. However, the estimate can not be applied directly to compensate the disturbances since here the disturbance acts via different channel to the control input. The mainly contribution of this paper lies in that a disturbance compensation vector is investigated for the
DOBC to attenuate the disturbances from the output channel asymptotically. Finally, a composite control method combining a feedback part based on linear quadratic regulator (LQR) and a feedforward part based on state-space DOB is proposed for the MAGLEV system. The proposed method provides a concise and practical approach for general nonlinear systems subject to lumped “mismatching” disturbances.

Simulation studies are carried out and the results show that the proposed new DOBC method provides appropriate disturbance rejection and has robustness against model uncertainties. The remaining of the paper is organized as follows. The dynamic model of the MAGLEV system is presented in Section II. In Section III, design of a state-space disturbance observer is presented first, and then the problem formulation follows. In Section IV, a new DOBC method is investigated for the MAGLEV system. Simulation studies are carried out in Section V. The conclusions are finally given in Section VI.

II. DYNAMIC MODEL OF THE MAGLEV SYSTEM

A. Nonlinear Model

The complete nonlinear model for the MAGLEV suspension system is given by [5],

\[
B = K_b \frac{I}{G},
\]

\[
F = K_f B^2,
\]

\[
\frac{dI}{dt} = \frac{V_{\text{coil}} - IR_c + \frac{N_c A_p K_b}{G_c} (dz_c - dZ)}{\frac{N_c A_p K_b}{G_c} + L_c},
\]

\[
\frac{d^2Z}{dt^2} = g - \frac{K_f I^2}{M_s G_c^2},
\]

\[
\frac{dG}{dt} = \frac{dz_c}{dt} - \frac{dZ}{dt},
\]

where variables \(I\), \(z_1\), \(Z\), \(\frac{dz_c}{dt}\), \(\frac{dZ}{dt}\), \(G\), \(B\) and \(F\) denote the current, the rail position, the electromagnet position, the rail vertical velocity, the electromagnet vertical velocity, the air gap, the flux density and the force, respectively. Signal \(V_{\text{coil}}\) is the voltage of the coil. Other symbols in Eqs. (1)-(5) are system parameters listed in Table I.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Meaning</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>(M_s)</td>
<td>Carriage Mass</td>
<td>1000kg</td>
</tr>
<tr>
<td>(F_o)</td>
<td>Nominal force</td>
<td>9810 N</td>
</tr>
<tr>
<td>(G_o)</td>
<td>Nominal air gap</td>
<td>0.015 m</td>
</tr>
<tr>
<td>(R_c)</td>
<td>Coil’s Resistance</td>
<td>10Ω</td>
</tr>
<tr>
<td>(B_o)</td>
<td>Nominal flux density</td>
<td>1T</td>
</tr>
<tr>
<td>(L_c)</td>
<td>Coil’s Inductance</td>
<td>0.1H</td>
</tr>
<tr>
<td>(I_o)</td>
<td>Nominal current</td>
<td>10A</td>
</tr>
<tr>
<td>(N_c)</td>
<td>Number of turns</td>
<td>2000</td>
</tr>
<tr>
<td>(V_o)</td>
<td>Nominal voltage</td>
<td>100V</td>
</tr>
<tr>
<td>(A_p)</td>
<td>Pole face area</td>
<td>0.01 m²</td>
</tr>
</tbody>
</table>

B. Linearized MAGLEV Suspension Model

The linearization of the MAGLEV suspension is based on small perturbations around the operating points. The following definitions are used in which the lower case letters define a small variation around the operating point and the subscript 'o' refers to the operating condition.

\[
B = B_o + b_1
\]

\[
F = F_o + f\
\]

\[
I = I_o + i
\]

\[
G = G_o + (z_t - z)
\]

\[
V_{\text{coil}} = V_o + u_{\text{coil}}
\]

\[
Z = Z_o + z
\]

The linearized state-space equation of the MAGLEV suspension model is expressed as

\[
\begin{align*}
\dot{x} &= Ax + Bu + Bd, \\
y &= Cx,
\end{align*}
\]

where the states are the linearized current, vertical electromagnet velocity and air gap, i.e., \(x = [\dot{I} \ z_t (z_t - z)]^T\), the input \(u = u_{\text{coil}}\) is the voltage, the track input \(d = \dot{z}_t\) is the rail vertical velocity. The controlled variable is selected as the variation of the air gap, i.e., \(y = z_t - z\). The detailed linearization procedure can be seen in [5], here we give the state matrix \(A\), the input matrix \(B_u\), the disturbance matrix \(B_d\).

\[
A = \begin{bmatrix}
-\frac{B_r}{L_c + K_b N_c A_p G_o} & -\frac{K_b N_c A_p I_o}{G_o^2 (L_c + K_b N_c A_p G_o)} & 0 \\
2K_f I_o^2 & 0 & 2K_f I_o^2 \\
0 & -1 & 0
\end{bmatrix},
\]

\[
B_u = \begin{bmatrix}
1 \\
\frac{1}{L_c + K_b N_c A_p G_o} \\
0
\end{bmatrix},
\]

\[
B_d = \begin{bmatrix}
\frac{K_b N_c A_p I_o}{G_o^2 (L_c + K_b N_c A_p G_o)} \\
0 \\
1
\end{bmatrix}.
\]
III. PRELIMINARY AND PROBLEM FORMULATION

A. State-Space Disturbance Observer

The main task of this subsection is to design an observer to estimate the system disturbances. Consider a linear system with lumped disturbances presented as

\[
\begin{align*}
\dot{x} &= Ax + Bu + B_l d_i, \\
y &= Cx.
\end{align*}
\]

(16)

Assumption 1: Suppose that the lumped disturbances \(d_i\) varied slowly relative to the observer dynamics, i.e., \(\dot{d}_i \approx 0\).

Remark 1: The results in this paper are based on Assumption 1. However, some work in literature points out that the estimate of DOB can track the disturbances lumped disturbances if the observer gain matrix \(L\) is chosen such that \(-LB_{id}\) are stable matrix, i.e., all eigenvalues of the matrix \(-LB_{id}\) have negative real part.

Proof: The disturbance estimation error of the DOB (17) is defined as

\[
e_{d} = \hat{d}_i - d_i,
\]

(18)

Considering Assumption 1, combine (16), (17) and (18) gives

\[
\begin{align*}
\dot{e}_d &= \hat{d}_i - d_i \\
&\approx \hat{p} + L\hat{x} \\
&= -LB_{id}\hat{d}_i - L(Ax + Bu) \\
&\quad + L(Ax + B_u u + B_{id}d_i) \\
&= -LB_{id}(\hat{d}_i - d_i) = -LB_{id}e_d.
\end{align*}
\]

(19)

Since all eigenvalues of matrix \(-LB_{id}\) are in the left half of the complex plane, Eq. (19) is asymptotically stable. This means that the estimate of DOB can track the disturbances asymptotically.

B. Problem Formulation

Note that in real engineering practice, besides external disturbances, model uncertainties including parameter perturbations, unmodeled nonlinear dynamics always bring about undesirable effects on linear control systems. In this work, the lumped disturbances consisting of both external disturbances and internal disturbances caused by model uncertainties are considered. The complete MAGLEV suspension system (12)-(15) can be presented as Eq. (16), where \(B_{id} = I_{1 \times 3}, d_i = [d_1, d_2, d_3]^T, \) and \(d_i (i = 1, 2, 3)\) denotes the lumped disturbance within the channel of state \(x_i\).

It can be observed from Eq. (16) that the disturbances enter the system with different channels from that of the control input and the “mismatching” disturbances happen. In previous literatures, the DOB methods are only focusing on the case of “matching” disturbances, i.e., the lumped disturbances \(d\) enter the system with the same channels of the control inputs. Precisely speaking, the “matching” case of disturbances means that the following two conditions are satisfied: 1) the control inputs \(u\) and the lumped disturbances \(d_i\) have the same dimension, and 2) in Eq. (16), \(B_u = B_{id}\). These conditions have constrained the application of DOB strategies to more general controlled plants.

Remark 2: Note that the DOB is applicable for the case of “mismatching” disturbances. However, the estimate of DOB cannot be used to compensate the disturbances directly because the disturbances are not in the same channels with the control inputs. The details are illustrated by the following example.

Considering a simple system expressed as

\[
\begin{align*}
\dot{x}_1 &= x_2 + d, \\
\dot{x}_2 &= x_1 + x_2 + u, \\
y &= x_1.
\end{align*}
\]

(20)

For system (20), the estimate \(\hat{d}\) of the real disturbance \(d\) can be obtained by DOB. However, if the composite control law is designed as \(u = K_d x - \hat{d}\) (where \(K_d\) is the feedback control gain) which is employed in all previous literatures regarding DOBC methods, we can find that the disturbance compensation design has nothing meaningful in this case because the disturbance can neither be attenuated from the state equations nor from the output channel.

It should be pointed out that the “mismatching” disturbances cannot be attenuated from the state equations generally. In this paper, based on the disturbance estimate of DOB, we design the composite control law as \(u = K_d x + K_d d\) and attempt to find an appropriate \(K_d\) to assure that the disturbances can be removed from the output channel finally. This method largely extends the application fields of the DOBC strategy.

A general DOBC design procedure for system (16) subjecting to “mismatching” disturbances is considered and given as follows:

1) Design a feedback controller to achieve stability without considering the disturbances.
2) Design a linear state-space disturbance observer to estimate the “mismatching” disturbances.
3) Design a disturbance compensation gain vector to achieve desired specification.
4) Integrating the feedback controller and the feedforward compensation term to formulate the composite DOBC.

<table>
<thead>
<tr>
<th>TABLE II</th>
<th>CONSTRAINTS FOR MAGLEV SUSPENSION SYSTEM</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constraints</td>
<td>Value</td>
</tr>
<tr>
<td>Maximum air gap deviation, ((z_t - z)_{p})</td>
<td>(\leq 0.005m)</td>
</tr>
<tr>
<td>Maximum input coil voltage, ((u_{coil})_{p})</td>
<td>(\leq 300V/(3I_{2}R_c))</td>
</tr>
<tr>
<td>Settling time, ((t_s))</td>
<td>(\leq 3s)</td>
</tr>
<tr>
<td>Air gap steady state error, ((z_t - z)_{s,e})</td>
<td>(= 0)</td>
</tr>
</tbody>
</table>
IV. DISTURBANCE OBSERVER BASED CONTROL FOR THE MAGLEV SYSTEM

A. Feedback Control Design

Actually, as for the new composite DOBC method, any feedback controller which can stabilize system (16) in the presence of disturbances are available. Here we choose the classical linear quadratic regulator (LQR). The penalty matrix $Q$ and $R$ in the cost function of LQR are selected as

$$Q = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad R = 0.1. \quad (21)$$

B. Stability Analysis of The Closed-Loop System

Different from those of all previous DOBC methods, our new DOBC control law for system (16) is designed as

$$u = K_d \dot{x} + K_d \dot{d}_t. \quad (22)$$

Combining system (16) with the DOBC law (22) and disturbance observer (17)-(19), the closed-loop system is obtained as

$$\begin{aligned}
\dot{x} &= \left[ A + B_u K_x \right] \dot{x} + \left[ B_u + B_d K_d \right] \dot{d}_t \\
\dot{e}_d &= \left[ A + B_u K_x \right] e_d + \left[ B_u + B_d K_d \right] d_t.
\end{aligned} \quad (23)$$

**Remark 3:** Eq. (23) shows that the disturbance observer can be separately designed from that of the feedback control part. This means that the disturbance observer can estimate the disturbances accurately for any $x \in \mathbb{R}^3$.

Since the lumped disturbances vary slowly, here we suppose that $d_t$ will not result in instability of the closed-loop system. Stability of the closed-loop system can be guaranteed by choosing asymptotically stable observer gain matrix $L$ and stabilized feedback gain $K_d$.

C. Design of The Disturbance Compensation Gain Vector

The main contribution of this work is investigating how to design the disturbance compensation gain vector $K_d$ such that the effects caused by the “mismatching” disturbances can be attenuated from the output channel asymptotically.

**Theorem 2:** Presume that disturbances in system (16) satisfy Assumption 1. Considering the general system (16) under the new designed DOBC law (22) consisting of stabilized feedback part $K_d \dot{x}$ and the disturbance compensation term $K_d \dot{d}_t$ based on the disturbance observer (17) (with appropriate chosen gain matrix $L$ assuring Eq. (19) is asymptotically stable), the disturbance can be attenuated from the output channel asymptotically if the disturbance compensation gain vector is selected as

$$K_d = \left[ C \left( A + B_u K_x \right)^{-1} B_u \right]^{-1} \times C \left( A + B_u K_x \right)^{-1} B_{ld}. \quad (24)$$

**Proof:** Substituting the control law (22) into system (16), the state is expressed as

$$x = \left( A + B_u K_x \right)^{-1} \left[ \dot{x} - B_u K_d \dot{d}_t - B_{ld} d_t \right]. \quad (25)$$

Combining (16), (24) and (25), gives

$$y = \left( A + B_u K_x \right)^{-1} \dot{x} + C \left( A + B_u K_x \right)^{-1} B_{ld} \left( \dot{d}_t - d_t \right). \quad (26)$$

Considering $\dot{x}(\infty) = 0$ in steady state, Eq. (26) is then reduced to

$$y(\infty) = \left( A + B_u K_x \right)^{-1} B_{ld} e_d(\infty). \quad (27)$$

The proof completes by using the result of Theorem 1. \(\square\)

**Remark 4:** Note that the disturbance compensation gain vector $K_d$ in (24) is a general case and suitable for both “matching” and “mismatching” disturbances. In “matching” case, i.e., $B_u = B_{ld}$, it can be obtained from (24) that the disturbance compensation gain vector is reduced to $K_d = -1$ which is the particular form in previous literatures.

In our work, the observer gain matrix in DOB (17) is selected as

$$L = \begin{bmatrix} 15 & 0 & 0 \\ 0 & 15 & 0 \\ 0 & 0 & 15 \end{bmatrix}. \quad (28)$$

The control structure of the proposed DOBC for the nonlinear MAGLEV suspension system is shown in Fig. 1.

Based on (21), the feedback LQR gain is obtained as $K_x = [-61 - 591 40061]$. The disturbance compensation gain vector can also be calculated by Eq. (24), gives as $K_d = [-2.1 36.0 742.2]$.

V. SIMULATION STUDIES

In this section, both external disturbances and model uncertainties are considered to show the effectiveness of the proposed new DOBC method.

A. External Disturbance Rejection Performance

The main external disturbances in MAGLEV system are the deterministic inputs to a suspension for the vertical direction. Such deterministic inputs are the transitions onto track gradients. In this paper, the deterministic input components
considered are referred to [5] and shown in Fig. 2. They represent a gradient of 5% at a vehicle speed of 15 m/s and an allowed acceleration of 0.5 m/s$^2$ while the jerk level is 1 m/s$^3$.

The response curves of both the output and input of the suspension system under the new DOBC method are shown in Fig. 3 by solid lines. Response curves of the corresponding states are shown in Fig. 4 by solid lines.

It can be observed from Fig. 3(a) that the maximum air gap deviation is less than 0.006 m, the settling time is shorter than 2.2 s and there is no steady-state error. All of these performances satisfy the design requirements listed in Table II. As shown in Fig. 3(b), the maximum input voltage in such case is about 35 V. Response curves in Fig. 4 show that both the current and the vertical electromagnet velocity vary smoothly and approach to the desired equilibrium points quickly. The results demonstrate that the proposed new DOBC method has achieved appropriate performance in rejecting such practical disturbances.

B. Robustness Against Load Variation

In this part, the load variation of the MAGLEV suspension is considered. The suspension has to support the large mass of the vehicle as well as the load (weight of the passengers) which can vary up to 40% of the total mass of the vehicle. This is a considerable variation of the total mass and may result in undesirable performance. To this end, the robustness against load variations should be taken into account to ensure performance and stability for a fully laden or unladen vehicle. For testing, we assume that the load variation is up to 25% of the total vehicle mass, i.e., the load can vary from 1000 kg to 1250 kg for a fully unladen and laden vehicle, respectively. The details of load variation are shown in Fig. 5.

The robustness against such case of load variation can be seen in Figs. 3 and 4 by dashed lines. It can be observed from Fig. 3(a) that the maximum air gap deviation is less than 0.006 m and there is still no steady-state error. Fig. 3(b) shows that the magnitude of the coil voltage is within the allowable region constrained in Table II. It also can be found in Fig. 4 that all the states vary smoothly. Test results in this subsection manifest that the new DOBC method obtains appropriate performance of robustness to load variation.

VI. CONCLUSION

A novel proposed disturbance observer based control (DOBC) method is utilized to assist on the disturbance attenuation problem in MAGLEV suspension system in this paper. The disturbance under consideration includes both external disturbances and internal disturbances caused by nonlinear unmodeled dynamics (neglected nonlinearities during linearization) and parameter perturbations (caused by load variation). In addition, the disturbance here is classified a mismatching case, i.e., the disturbance act via different channels to the control input. Previous DOBC methods did not handle such mismatching disturbances. Via a chosen disturbance compensation gain vector, a new DOBC method has been proposed for the MAGLEV system with lumped mismatching disturbances. The simulation results have demonstrated that the proposed method obtains the
Fig. 4. Response curves of the states in the presence of deterministic track input: (a) the current, $i$, (b) the vertical electromagnet velocity, $\dot{z}$.

Fig. 5. Curve of the load variation.

required disturbance rejection performance as well as appropriate robustness against load variation when controlling the given nonlinear MAGLEV system.

REFERENCES